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THEORETICAL AND COMPUTATIONAL STUDIES OF A RECTANGULAR FINITE WING OSCILLATING IN PITCH AND HEAVE

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Abstract. This paper explores the extent of validity of Unsteady Lifting Line Theory (ULLT) for finite rectangular wings undergoing pitch-heave oscillation in the low Reynolds number regime. Sclavounos' ULLT is compared to Theodorsen-based strip theory and high fidelity Computational Fluid Dynamics (CFD). Pitch and heave kinematics of different frequencies for wings with varying aspect ratio are considered. A pitch-heave equivalence theory is also tested by seeking equivalent pitching motions for given heave kinematics. It was found that the ULLT successfully modelled finite-wing effects up to high frequencies. The pitch-heave equivalence theory was found to be effective.

1 INTRODUCTION

The study of oscillating finite wings is becoming increasingly important in light of advances in high aspect-ratio, flexible aircraft, novel energy harvesting mechanisms based on oscillating wings and micro air vehicles with flapping wings.

To analyse the loads resulting from the unsteady flow, engineers have turned to numerically expensive methods such as vortex particle methods and computational fluid dynamics (CFD). Despite advances in computing power, these methods are time consuming and expensive. They are unsuitable for real-time simulation, control or use in large-scale optimisations.

Lifting-line theory provides an alternative low-order means by which finite wing effects can be modelled. Originally implemented by Prandtl and later justified by Van Dyke [1], the lifting line model is one of the simplest asymptotic models in aerodynamics. The model is valid on the assumption that the span length scale b is significantly larger than the chord scale c. Naturally, an unsteady extension to the theory was sought.

Research has focused on the frequency domain. The resulting Unsteady Lifting-Line Theories (ULLT) are 3D analogues of Theodorsen's theory [2]. A significant challenge for those devising such theories has been achieving validity over the entire frequency

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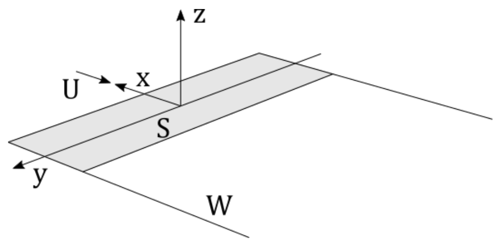


Figure 1: Problem coordinate system

range. Cheng et al. [3] devised a framework to describe the relationship between an ULLT and the frequency of oscillation. It was stated that for a wing of chord scale c and span scale b harmonically oscillating with wake wavelength λ, five ranges of λ can be identified: the very low frequencies ($c \ll b \ll \lambda$), the low frequencies ($c \ll b = O(\lambda)$), the intermediate frequencies ($c \ll \lambda \ll b$), the high frequencies ($c = O(\lambda) \ll b$) and the very high frequencies ($\lambda \ll c \ll b$).

The first ULLT was that of James [4]. It was valid only for low frequencies. Ahmadi and Widnall [5] corrected errors in James' theory, extending it to spanwise flexible wings. Cheng and Murillo [6] produced a theory valid for oscillating swept wings, but only valid for intermediate frequencies. Sclavounos [7] produced a method valid up to high frequencies. Guermond and Sellier [8] produced a method capable of modelling curved, swept wings over the entire frequency domain.

Verification appears to have been a problem for ULLTs. While experimental data including forces histories is extensive for 2D cases, only limited investigation of the force history of 3D cases appears to have been undertaken. This is despite the volume of literature on the flow topology for such unsteady problems. Chiereghin et al. [9] investigate finite swept wings providing RMS values for force, and values for phase offset. Dong et al. [10] use CFD to analyse a elliptic oscillating 'fish fin', but the lift coefficient was a secondary concern.

The current study aims to examine the lift history of rectangular finite wings based on the SD7003 aerofoil for a small variety of chord reduced frequencies, span reduced frequencies and pitch and heave amplitudes. The results will be compared to strip theory based on Theodorsen's theory and the unsteady lifting line theory of Sclavounos .

The problem definition, theory and numerical methods will be introduced in Section 2. Section 3 will examine and discuss the results, followed by conclusions in Section 4.

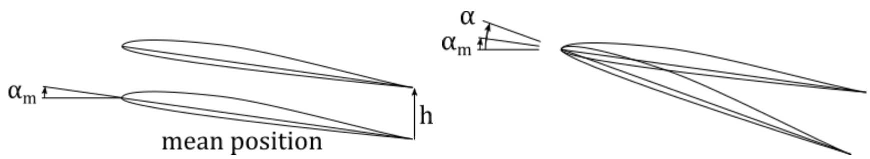
2 THEORY

2.1 Kinematics

A rectangular wing, shown in Figure 1, may be defined by its semispan d and semichord l. The wing undergoes harmonic oscillations in both pitch and heave whilst subject to a free stream of velocity U.

At a chord section of this wing, the heave displacement may be given as

$$h = \mathrm{Re}(h_m e^{i\omega t}) \tag{1}$$



(a) Plunge definition (b) Pitch definition

Figure 2: Kinematics definitions. Positive displacements shown.

where h_m is the complex heave amplitude, ω is the circular frequency of oscillation and t is time. This is graphically depicted in Figure 2(a).

The angle of attack $\alpha(t)$ may be given as

$$\alpha(t) = \alpha_m + \mathrm{Re}(\theta_m e^{i\omega t}) \tag{2}$$

where α_m is the mean angle of attack and θ_m is the complex amplitude of pitch oscillation. The direction of positive α is showing in Figure 2(b).

The heave and pitch displacements are constant across the span of the wing. This allows two reduced frequencies to be defined. Firstly, the chord reduced frequency

$$k = \frac{\omega l}{U} \tag{3}$$

and secondly the span reduced frequency

$$\nu = \frac{\omega d}{U} \tag{4}$$

2.2 Theodorsen's method

Theodorsen's method [2] has been widely used to evaluate the loads on a sinusoidally oscillating aerofoil.

Linearity is assumed. Hence, the 3D lift coefficient C_L can be decomposed into steady and unsteady parts:

$$C_L = C_{Lss} + C_{Lu} \tag{5}$$

where C_{Lss} and C_{Lu} are the steady state and unsteady 3D lift coefficients respectively.

The 2D unsteady component may be given as

$$C_{ltu} = \mathrm{Re}\left(\pi l \left[\frac{\dot{\theta}}{U} - \frac{la\ddot{\theta}}{U^2} - \frac{\ddot{h}}{U^2}\right] + 2\pi C(k)\left[\theta - \frac{\dot{h}}{U} + b\left(\frac{1}{2} - a\right)\frac{\dot{\theta}}{U}\right]\right) \tag{6}$$

where the subscript ltu indicates the unsteady lift coefficient of Theodorsen's theory, and

$$C(k) = \frac{H_1^{(2)}}{H_1^{(2)}(k) + iH_0^{(2)}(k)} \tag{7}$$

is the Theodorsen function. $H_1^{(2)}(x)$ and $H_0^{(2)}(x)$ are Hankel functions of the second kind.

To obtain a 3D unsteady lift coefficient based on Theodorsen's theory, we can use strip theory:

$$C_{Lu} = \frac{2}{S}\int_{-d}^{d} C_{ltu} l(y)\, dy \tag{8}$$

where S is the planform area of the wing.

2.3 Sclavounos' Method

Sclavounos models a straight planar wing and planar wake, with sinusoidal oscillation in heave or pitch about the midchord. A lifting-line theory is developed valid up to high frequencies ($k < O(1)$). Errors are of the $O(1/A)$ for an inner solution varying on length scale d. Though technically limited to wings with cusped tips, the LLT has been successfully applied when this is not the case.

The inner solution is similar to Theodorsen's method, resulting in identical result as the span reduced frequency tends to infinity.

The pitch and heave circulations are computed independently. Their respective lift coefficient contributions C_{Lsp} and C_{Lsh} can be summed to obtain the total unsteady lift coefficient C_{Lu}. The unsteady circulation at any point on the span is modelled as a complex amplitude, allowing the time factor ($e^{i\omega t}$) to be temporarily ignored. Normalised with the maximum pitch or heave displacement, the equation for circulation $\Gamma_j(y)$ is obtained as

$$\Gamma_j(y) - \frac{d_3(y)}{2\pi i\omega}\int_{-d}^{d} d\eta\, \Gamma'_j(\eta) K(y-\eta) = \begin{cases} d_3(y), & j=3 \\ d_5(y) - \frac{U}{i\omega} d_3(y), & j=5 \end{cases} \tag{9}$$

where the kernel K is defined as

$$K(y) = \frac{1}{2}\texttt{sgn}(y)\left[\frac{e^{-\frac{\omega|y|}{U}}}{|y|} - \frac{i\omega}{U}E_1\left(\frac{\omega|y|}{U}\right) + \frac{\omega}{U}P\left(\frac{\omega|y|}{U}\right)\right] \tag{10}$$

where $E_1(x)$ is the exponential integral and

$$P(y) = \int_1^\infty dt\, e^{-yt}\left[\frac{\sqrt{t^2-1}-t}{t}\right] + i\int_0^1 dt\, e^{-yt}\left[\frac{\sqrt{1-t^2}-1}{t}\right] \tag{11}$$

$j = 3$ and $j = 5$ refer to the heave and pitch respectively.

d_3 and d_5 represent the vortex strength required to satisfy the velocity boundary condition on the wing normalised for the displacement and pitch amplitudes respectively.

$$d_3 = \frac{4Ue^{-\frac{i\omega l}{U}}}{iH_0^{(2)}(k) + H_1^{(2)}(k)} \tag{12}$$

and

$$d_5(y) = -\frac{l d_3(y)}{2}. \tag{13}$$

The vorticity distribution gives rise to an interaction function

$$F_j = \begin{cases} 1 - \frac{\Gamma_3(y)}{d_3(y)}, & j = 3 \\ \frac{d_5(y) - \Gamma_5}{d_3(y)} - \frac{U}{i\omega}, & j = 5 \end{cases} \tag{14}$$

which tends to zero for the 2D case.

The lift coefficients for heave oscillation is

$$C_{Lsh} = \mathrm{Re}\left(\frac{i\omega h_m e^{i\omega t}}{U}\left[-\frac{4\pi}{S}\int_{-d}^{d} dy\, C(k) l(y)(1 - F_3(y)) - i\frac{a_{33}\omega}{SU}\right]\right) \tag{15}$$

where a_{33} is the heave added mass of the planform area S. This may be obtained using a strip theory approach

$$\int_{-d}^{d} \frac{2\pi\omega l^2}{U}\, dy = \frac{a_{33}\omega}{SU} \tag{16}$$

The coefficient of lift for pitch may be obtained as

$$C_{Lsp} = \mathrm{Re}\left(\frac{i\omega\theta_m e^{i\omega t}}{U}\left[\frac{2\pi}{S}\int_{-d}^{d} dy\, l\left[C(k)\left(l + \frac{2U}{i\omega} + 2F_5\right) + l\left(1 + \frac{i\omega F_5}{U}\right)\right]\right]\right) \tag{17}$$

2.4 Pitch-heave equivalence

McGowan et al. [11] presented pitch-heave matching as a novel way to test the validity of a 2D theory in different regimes. This can be done in 3D by matching the resultant forces.

$$C_{Lsp} = C_{Lsh} \tag{18}$$

The use of a rectangular wing permits a simplification. Equation (9) may be rewritten in purely terms of $d_3(y)$ using Equation (13):

$$d_5(y) - \frac{U}{i\omega} d_3(y) = -d_3(y)\left(\frac{l(y)}{2} - \frac{U}{i\omega}\right)$$

meaning that, for a rectangular wing,

$$F_5(y) = -\left(\frac{l}{2} + \frac{U}{i\omega}\right) F_3(y). \tag{19}$$

This allows us to simplify Equation 18 to

$$\theta_m I_{LspR} = h_m I_{LshR} \tag{20}$$

where, if we already have already computed the plunge interaction function $F_3(y)$

$$I_{LshR} = 2d\left(C(k) + \frac{ik}{2}\right) - C(k)\int_{-d}^{d} F_3\, dy \tag{21}$$

and

$$I_{LspR} = \left(\frac{l}{2} + \frac{U}{i\omega}\right)\left(2C(k) + \frac{li\omega}{U}\right)\int_{-d}^{d} F_3(y)\,dy - 2d\left(C(k)\left(l + \frac{2U}{i\omega}\right) + l\right) \tag{22}$$

In cases where the pitch-interaction function has already been computed, the substitution given by Equation (19) may be used. It should be noted that the strip theory approximation for added mass given in Equation (16) was used.

The kinematics of the problem may be extended to a pitch axis other than the midchord. For small displacements and a wing pitching a distance a in front of the midchord

$$\theta_m(C_{Lsp}/\theta_m + aC_{Lsh}/h_m) = C_{Lsh} \tag{23}$$

If a wing with no spanwise interaction is considered, $F_3(y) = F_5(y) = 0$. This yields an expression for two dimensional pitch-heave matching. The expression

$$\theta_m = -2h_m \frac{2C(k) + ik}{1 + C(k)\left(1 + \frac{2}{ik}\right) - 2a\left(C(k) + \frac{ik}{2}\right)} \tag{24}$$

is equivalent to that given by McGowan et al.[11].

2.5 Numerical methods

High-fidelity 3D computations of unsteady fluid dynamics are performed at Reynolds number of $10,000$ using the open-source CFD toolbox OpenFOAM. A body-fitted computational mesh is moved in accordance with prescribed rate laws, and the time-dependent incompressible Navier-Stokes are solved using a finite-volume method. A second-order backward implicit scheme is adopted to discretize the transient terms, while second-order, limited Gaussian integration schemes are used for the gradient, divergence and Laplacian terms. The pressure implicit with splitting of operators (PISO) algorithm is employed to achieve pressure-velocity coupling. The Spalart-Allmaras (SA) turbulence model [12] is used for turbulence closure. The SA model is chosen for this problem because of extensive previous experience in applying it successfully for unsteady, separated and vortex-dominated flows at $Re = 10,000$ such as those considered in this research [11, 13]. The trip terms in the original SA model are turned off, and for the low Reynolds number cases considered in this research, the effects of the turbulence model are confined to the shed vortical structures and wake.

A close-up of a representative computational mesh is shown in Figure 3. The chord length $c = 3$ inches. The O-mesh has 120 cells chordwise , with increased resolution near the leading and trailing edges. The average spanwise spacing on the wing is $c/72$, with increased resolution at the wingtip. The spanwise domain extends 4 chord lengths beyond the wingtip with an average spacing of $c/36$ in this region. In the wall-normal direction, cell spacing begins at 4×10^{-5} m next to the wall ($y+ < 1$) and is constrained to maximum spacing of $c/100$ up to a distance of $1.5c$ away from the wing. From thereon, a coarser mesh extends to $14c$ from the wing with a maximum spacing of $c/5$. The simulations were carried out at a free stream velocity $U = 0.1312$ m/s and kinematic viscosity 10^{-6} m^2/s.

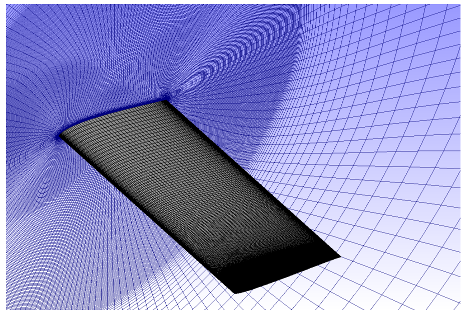


Figure 3: Close-up of computational mesh.

3 RESULTS AND DISCUSSION

The extent of validity of ULLT method was tested by violating some of the assumptions on which the model is based. Aspect ratio (AR), chord reduced frequency and oscillation amplitude were manipulated.

For each case, a rectangular wing with an SD7003 section is considered. The mean pitch angle α_m is 4 degrees. All heaving motions are purely sinusoidal, with any phase differences given relative to this.

Initially, a set baseline cases of wings oscillating in pure heave at high frequency and low aspect ratio are studied. The ULLT is valid for these cases. Next, we examine chord reduced frequencies where the ULLT is technically invalid. Third we examine a set of cases where the ULLT is invalid due the effects of large amplitude plunge oscillation. Here, flow separation and vortex shedding are expected. Finally the method for pitch-heave matching, given in Section 2.4 is tested, also allowing evaluation of the ULLT for pure pitching cases.

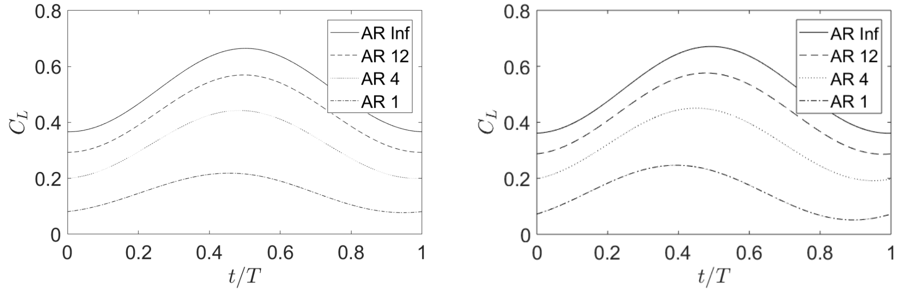
A quantitative summary of the results can be found in the appendix. The results of the CFD are given in Table 1, and the errors due to the use of a Theodorsen based strip theory (equivalent to an infinite aspect ratio wing) and the ULLT given in Table 2. Where C_L is calculated, the time averaged lift is taken as C_{Lss} for both the ULLT and Theodorsen theories.

3.1 Baseline cases

This case studies wings oscillating in heave at high frequency and low amplitude, such that $k = 0.393$ and $|h_m/c| = 0.05$. The ULLT is expected to be valid for this case.

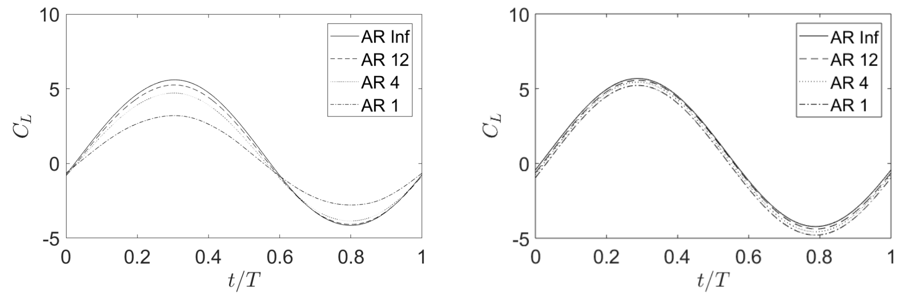
Figure 4(a) shows the CFD results. The loss of both lift amplitude and average lift coefficient is evident with decreasing aspect ratio.

Figure 4(b) shows the theoretical finite wing results. Whilst Theodorsen theory (infinite aspect ratio) provides a good approximation for high AR problems, finite wing effects become important for AR 4 and AR 1. For these cases, the ULLT provides a far more accurate approximation of lift amplitude. At AR 1, where the error of the ULLT is theoretically of $O(1)$, Theodorsen theory exhibits an error of over 100% . The error of the ULLT, whilst large, is comparatively excellent.



(a) CFD (b) Theory

Figure 4: Coefficient of lift comparison for baseline cases.



(a) CFD (b) Theory

Figure 5: Coefficient of lift comparison for very high frequency cases.

As aspect ratio decreases, the phase of the CFD results and the ULLT results lags behind the CFD result for infinite AR. The lag of the ULLT is however greater than that of the CFD.

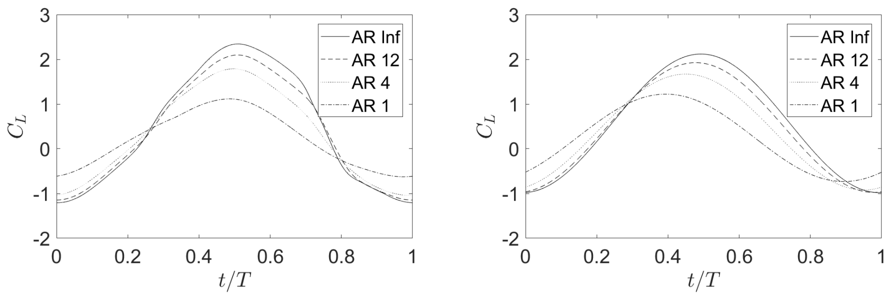
3.2 Study: Very high frequency

This section considers a wing heaving at a chord reduced frequency $k = 3.93$ and an amplitude $|h_m/c| = 0.05$. The Sclavounos ULLT is theoretically invalid due to the high frequency.

Figure 5(a) shows the results from CFD. Compared to the baseline case, the relative lift amplitude loss as aspect ratio is decreased is lessened. This is owing to the dominant contribution of apparent mass to lift at this frequency.

The lift coefficient predicted by the ULLT is shown in Figure 5(b). The predictions for all aspect ratios are very close, and contrary to expectation, as the aspect ratio is decreased the ULLT predicts slightly increased lift. At very high frequencies, the ULLT results in a circulatory lift in antiphase with the dominant added mass effects. Hence, as the circulatory lift decreases with aspect ratio, the sum of the lifts increases.

Overall, the ULLT shows no accuracy advantage over the Theodorsen strip theory for very high frequencies.



(a) CFD (b) Theory

Figure 6: Coefficient of lift comparison for high amplitude cases.

3.3 Study: Large amplitude

This case is a large amplitude pure plunge modification of the baseline case. A chord reduced frequency of 0.393 at amplitude $|h_m/c| = 0.5$ was studied. Due to the large amplitude, flow separation and vortex shedding (observed in CFD), the ULLT is technically invalid.

The results of the CFD are shown in Figure 6(a). The effects of vortex shedding are prominent in the non-sinusoidal waveform. The lift amplitude losses with decreasing AR were similar to those in the baseline cases, but different to those in the very high frequency cases. Similar trends are observed in the ULLT results which are in surprisingly fair agreement with CFD.

The CFD lift amplitude result for infinite AR was greater than predicted by the Theodorsen theory. This is due to leading edge vortex shedding, which is not modelled in either Theodorsen's theory or the ULLT.

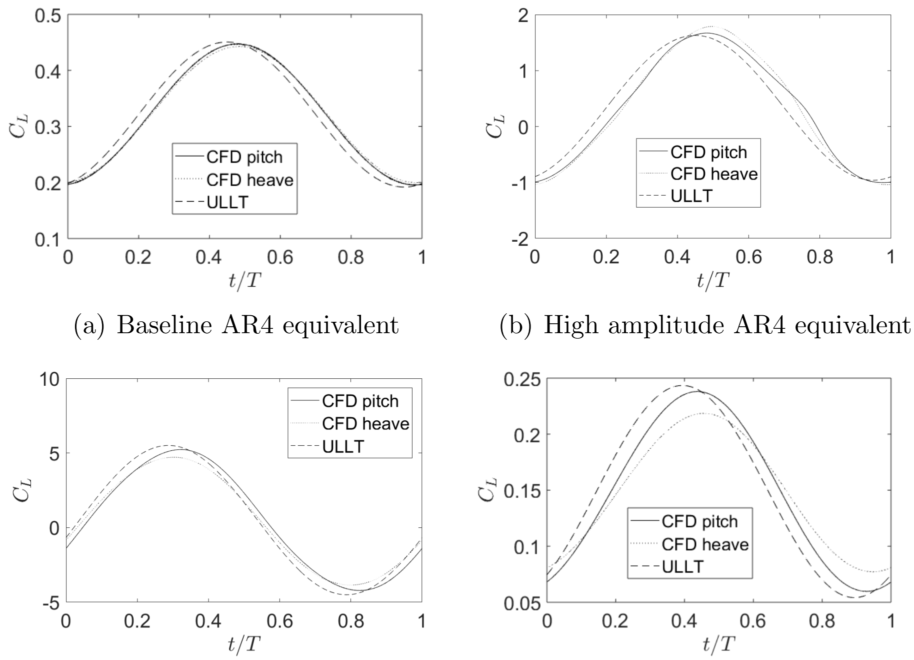
The effectiveness of the ULLT is surprising, with small errors even at low aspect ratio. This is partially due to the overestimate in the baseline cases being reduced due to an increase in lift caused by leading edge vortex shedding.

The vortex structures are 3D in nature, affecting some parts of the wing significantly more than others. This violates the assumption that the flow varies on the lengthscale d. In conjunction with the large amplitude of oscillation leading to a highly non-planar wake, the ULLT is theoretically not well suited to this problem. However, it proves remarkably robust.

3.4 Study: Pitch equivalence

Four pure leading edge pitching cases were constructed using the theory developed in Section 2.4. The $C_L(t)$ curves for these cases are shown in Figure 7.

Figure 7(a) is equivalent to the baseline AR4 heaving case. For both this pure pitching case and its heaving counterpart, the ULLT is valid. The kinematic phase lag was 18 degrees and the pitch amplitude was -5.4 degrees. The agreement of the pitching and the heaving CFD results was excellent. The ULLT also predicted the lift amplitude with good accuracy.



(a) Baseline AR4 equivalent

(b) High amplitude AR4 equivalent

(c) Very high frequency AR4 equivalent

(d) Baseline AR1 equivalent

Figure 7: Pure pitch cases equivalent to pure plunge cases.

Results for a pitch case equivalent to the large amplitude AR4 case are shown in Figure 7(b). A kinematic phase offset of 34.4 degrees and an pitch amplitude of -20.2 degrees was used. Despite the invalidity of the ULLT, the CFD cases were once again in good agreement.

A case theoretically equivalent to the high frequency AR4 case is shown in Figure 7(c). The kinematic phase offset was 18.4 degrees and the amplitude was -5.4 degrees. The error in the CFD is reasonably small and the ULLT predicts the lift amplitude with good accuracy.

Figure 7(d) show equivalent pitching case for the AR1 baseline case. A kinematic phase offset of 69.3 degrees and pitch amplitude of -2.4 degrees was used. Again, the difference between the CFD cases is small and the ULLT accurately predicts the lift amplitude of the pitch case.

In all cases the ULLT was in good agreement with the CFD in terms of amplitude, typically with better errors than found in pure heave. The difference in lift amplitudes for the CFD between equivalent pitching and heaving cases was remarkably good, often with smaller errors than ULLT in heave.

4 CONCLUSIONS

For a rectangular wing oscillating at high frequency it was found that the Sclavounos ULLT obtained better results than Theodorsen-based strip theory, successfully accounting for finite-wing effects. The ULLT was found to be robust beyond its theoretical assumptions, producing accurate results in high amplitude oscillation cases. Whilst the accuracy suffered at very low aspect ratio and at high reduced frequencies, it remained a significant improvement over Theodorsen-based strip theory. The ULLT was also found to be accurate for pitching problems. The success of the pitch-heave lift matching procedure also proved effective.

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APPENDIX

Kinematics	C_L amplitude (peak to peak)				C_L average			
	AR∞	AR12	AR4	AR1	AR∞	AR12	AR4	AR1
H1	0.298	0.277	0.243	0.141	0.516	0.431	0.321	0.149
H2	9.75	9.34	8.57	5.99	0.729	0.589	0.431	0.213
H3	3.55	3.24	2.83	1.74	0.569	0.475	0.374	0.245
P1			0.254	0.179			0.321	0.149
P2			9.47				0.503	
P3			2.67				0.334	

Table 1: CFD lift amplitudes and averages. H1, H2 and H3 indicate the baseline, very high frequency and high amplitude pure heave case kinematics respectively. P1, P2 and P3 indicate pure pitch cases generated to be equivalent to H1, H2 and H3 respectively.

Kinematics	Validity	Theodorsen amplitude error (%)				ULLT amplitude error (%)		
		AR∞	AR12	AR4	AR1	AR12	AR4	AR1
H1	Valid	4	12	27	114	5	6	38
H2	Invalid	3	6	15	65	6	17	71
H3	Invalid	-12	4	10	78	11	8	12
P1	Valid			33	129		2	9
P2	Invalid			8			6	
P3	Invalid			26			3	

Table 2: Error in theoretical predictions compared to CFD results. H1, H2 and H3 indicate the baseline, very high frequency and high amplitude pure heave case kinematics respectively. P1, P2 and P3 indicate pure pitch cases generated to be equivalent to H1, H2 and H3 respectively.

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